Density

$\rho = \frac{m}{V}$ — m: mass, V: volume

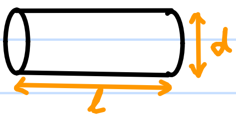
rho ρ is not p pee

- Density cannot be directly measured,
- It can be determined by measuring mass with a balance (not a scale!) and determining the volume

Determining volume:

Regular solid bodies measure necessary lengths, which depends on shape

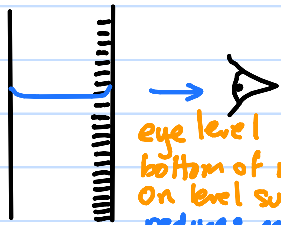
Cylinder:



$V = \frac{1}{4}\pi d^2 l$ $\left\{ r^2 = \left(\frac{1}{2}d\right)^2 = \frac{1}{4}d^2 \right\}$

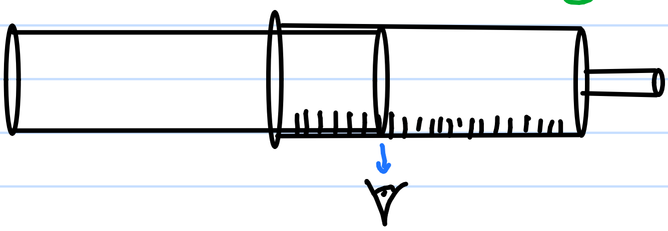
- measure diameter using micrometer ±0.01mm
- measure length using vernier calipers ±0.1mm or a ruler ±1mm

Liquids measure volume using measuring cylinder



eye level
bottom of meniscus
on level surface
reduces parallax error

Gases measure volume using gas syringe



must be horizontal

Irregular solid bodies

1. Measure volume of water in measuring cylinder
2. Place body in water (rigid body), make sure it is immersed
3. measure volume of water + body
4. Volume of body = combined volume — original volume of water

Alloys

contains two metals A & B

$V \neq V_A + V_B$ but $M = m_A + m_B$ (alloy volume; alloy mass)

$m_A = \rho_A V_A$ $\qquad m_B = \rho_B V_B$ $\qquad M = \rho V$ (alloy density)

so $M = \rho_A V_A + \rho_B V_B$

$$\rho = \frac{M}{V} = \frac{\rho_A V_A + \rho_B V_B}{V} = \rho_A \frac{V_A}{V} + \rho_B \frac{V_B}{V}$$

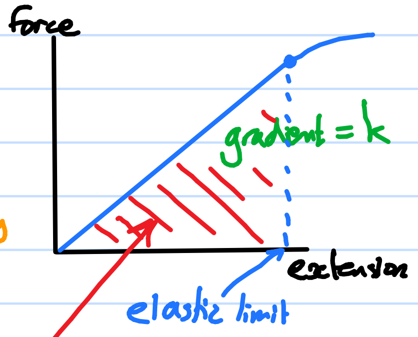
Springs

Force pulling spring

Hooke's law: $F \propto \Delta l \Rightarrow F = k\Delta l$

Spring constant (k), extension of spring (Δl)

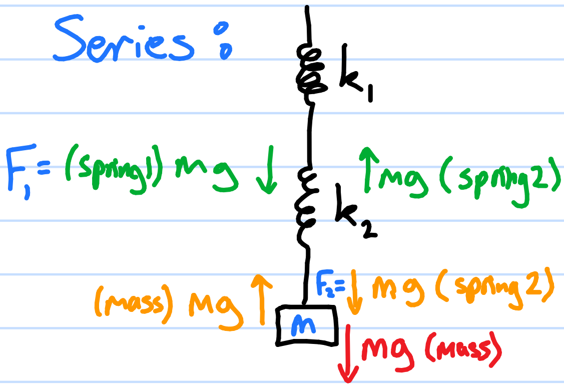
For extensions below the elastic limit



area = energy stored $= \frac{1}{2}F\Delta l$
$= \frac{1}{2}k(\Delta l)^2$

Combination of springs

Series:



Force on spring 1, force on spring network

$F_1 = F_2 = F$ if springs have negligable mass

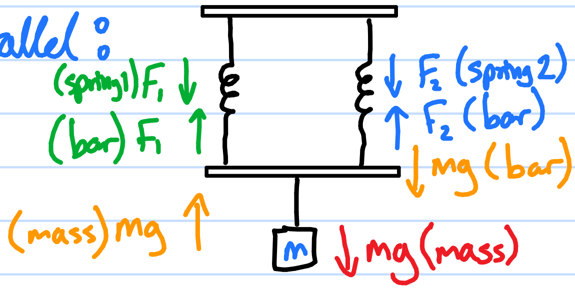
force on spring 2

$\Delta l = \Delta l_1 + \Delta l_2$ and $\Delta l = \frac{F}{k}$

total extension, combined spring contant

$$\frac{F}{k} = \frac{F}{k_1} + \frac{F}{k_2} \Rightarrow \boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}} \quad k = \left(k_1^{-1} + k_2^{-1}\right)^{-1}$$

Parallel:



$F_1 + F_2 = mg = F$

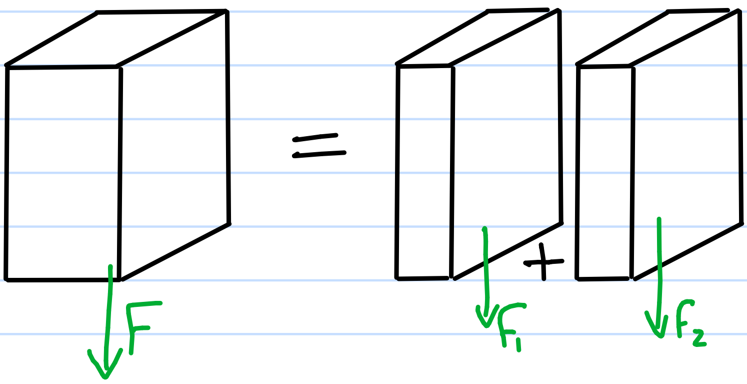
$F_1 = k_1\Delta l_1 \quad F_2 = k_2\Delta l_2$

$\Delta l = \Delta l_1 = \Delta l_2 \quad F = k\Delta l$

$$F = F_1 + F_2 \therefore k\cancel{\Delta l} = k_1\cancel{\Delta l_1} + k_2\cancel{\Delta l_2}$$

$$\boxed{k = k_1 + k_2}$$

Example

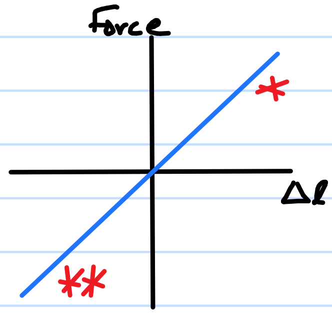


$F_1 + F_2 = F$

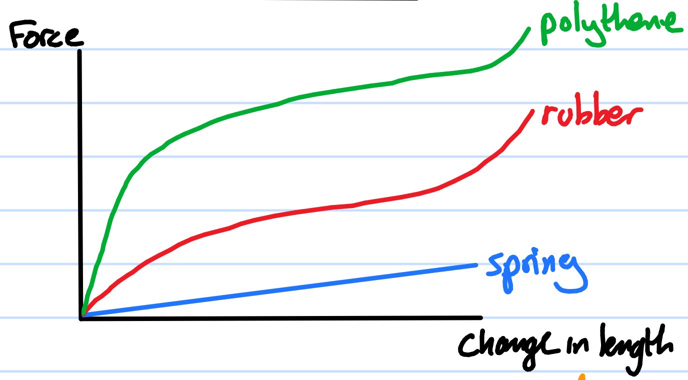
Deformation of solids

- Stretching → tension
- Squashing → compression
- Twisting → torsion
- Skewing → shear

* **



Different materials



Rubber deformed elastically
Polythene deformed plastically

Elastic deformation: when forces removed, return to original shape
Plastic deformation: permanent!

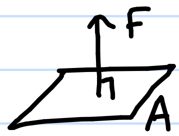
These graphs are geometry dependent.

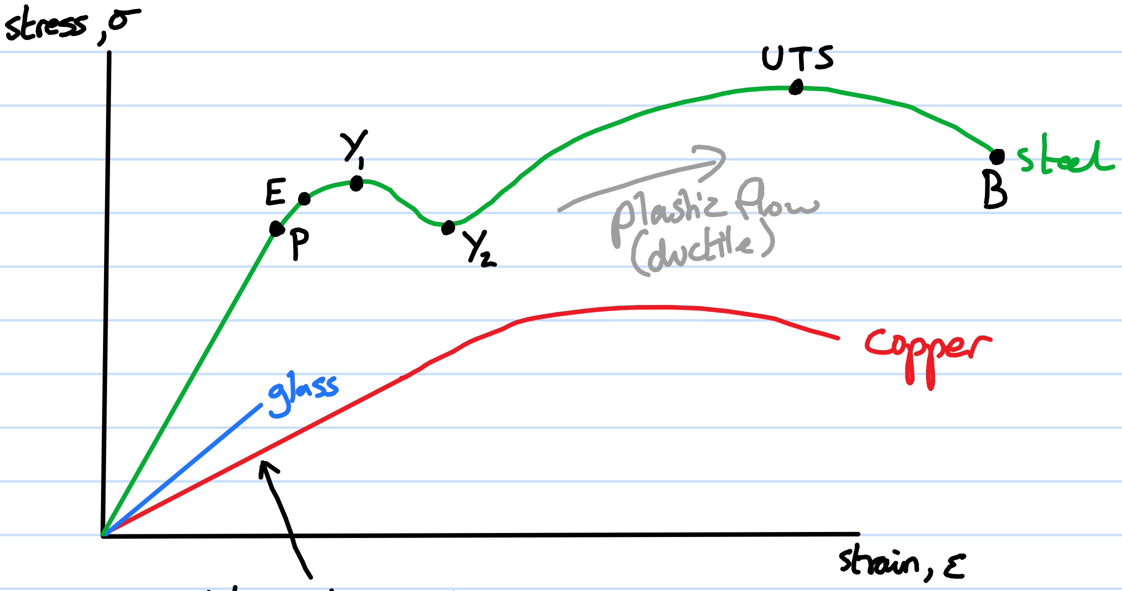
Tensile stress and tensile strain

Consider original length of a wire (l) for a given force F, I get extension $\Delta l = \frac{F}{k}$. If wire started twice as long ($2l$), for same force, we get $2\Delta l$ because k will halve.

We use tensile strain instead $\varepsilon = \frac{\text{extension}}{\text{original length}} = \frac{\Delta l}{l}$

Do the same thing for cross sectional area.
Instead of force, we use stress $\sigma = \frac{\text{tensile force}}{\text{cross sectional area}}$

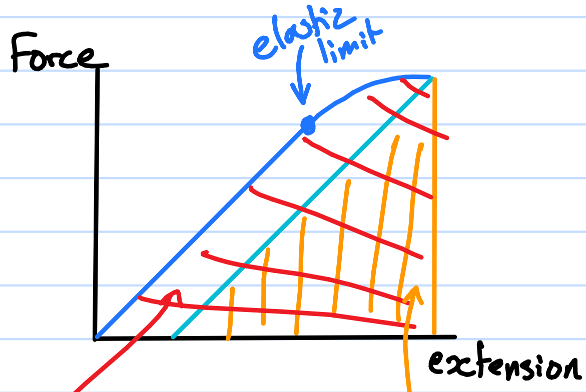




P – limit of proportionality
E – elastic limit
Y_1 – Yield point weakening
Y_2 – Yield point weakening
UTS – Ultimate tensile Stress strength
B – breaking point

Young's Modulus = gradient $= \frac{\sigma}{\varepsilon} = E$ (NOT ENERGY)

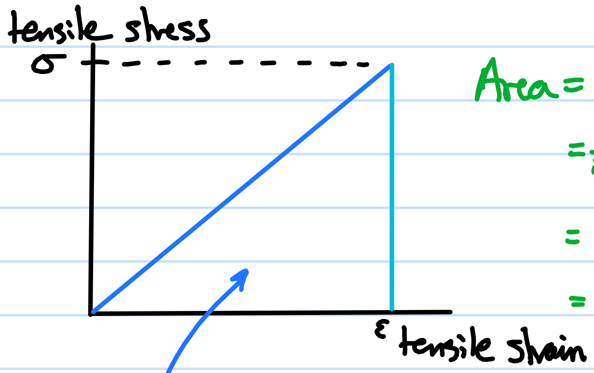
More about stress and strain



area = work done in extending

energy released when relaxed

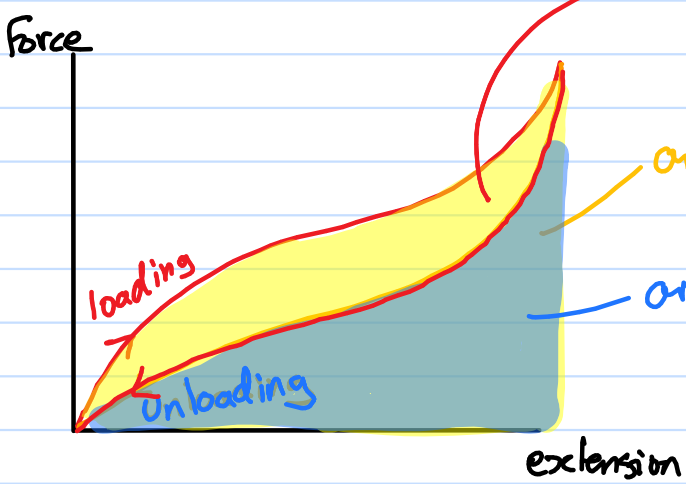
Difference is energy transferred to thermal energy stores



$$\text{Area} = \frac{1}{2}\sigma\varepsilon = \frac{1}{2}\frac{F}{A}\cdot\frac{\Delta l}{l} = \frac{1}{2}\frac{F\Delta l}{V} = \frac{W}{V}$$

Area = volumetric energy density $= \frac{W}{V}$

Rubber



area, $A_L - A_U$ = energy transferred to thermal energy stores i.e. wasted

area under loading curve, A_L = work done loading rubber

area under unloading curve, A_U = energy transferred when unloading i.e. strain energy stored

How to determine areas

- Count squares
- Every part square counts as a half square
- Multiply total squares by work done represented by area of 1 sq i.e. force × extension for one square