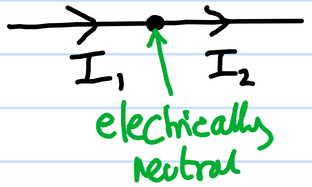
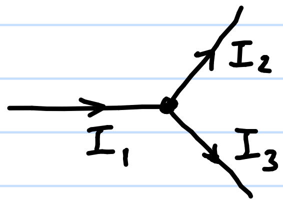
Circuit rules

Kirchhoff's first law: $\Sigma I = 0$
(Kirchhoff's current law) - KCL



$I_1 = I_2$ because of conservation of charge and charge does not accumulate at the junction ($I = \frac{Q}{t}$)



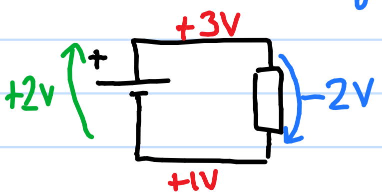
$I_1 = I_2 + I_3$

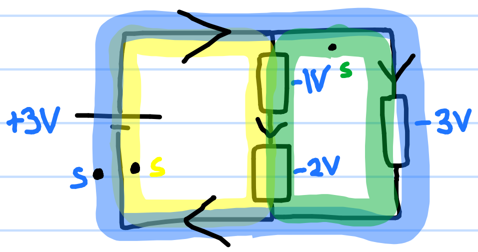
lets say current into a junction is +ve and current out of a junction is −ve.

$\Sigma I = I_1 + I_2 + I_3 = 0$

+ve −ve −ve

Kirchhoff's second law: $\Sigma V = 0$ (around a circuit loop)
(Kirchhoff's voltage law) - KVL





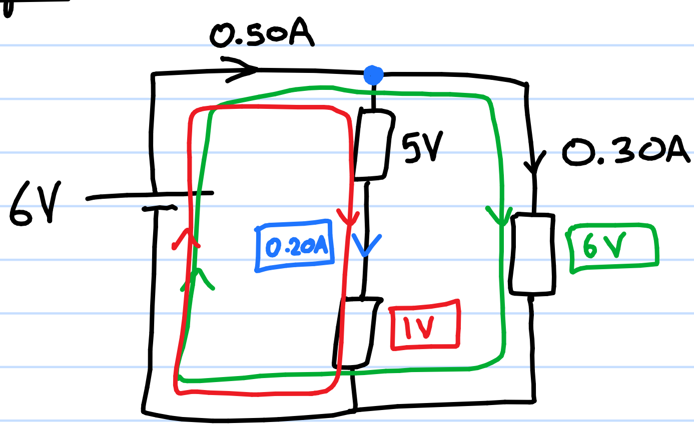
$3 - 1 - 2 = 0$

$3 - 3 = 0$

$-3 - (-2) - (-1) = 0$

Conservation of energy because ($V = \frac{W}{Q}$)

Example:



KI: $0.50 - 0.30 - I = 0$
$I = 0.20$

KII: $6 - V = 0$
$V = 6$

KII: $6 - 5 - V = 0$
$V = 1$

More about resistance

$P=VI \qquad V=IR \qquad I=\frac{V}{R}$

$P=I^2R \qquad P=V^2/R$

$P \propto R$ if I is constant

$P \propto 1/R$ if V is constant

- Electromotive force, emf ε is p.d. gained from power supply

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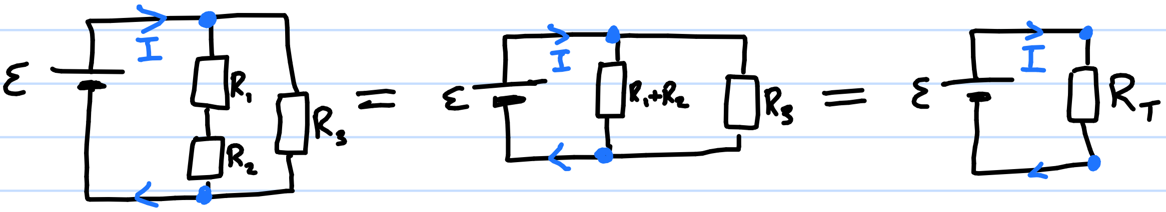
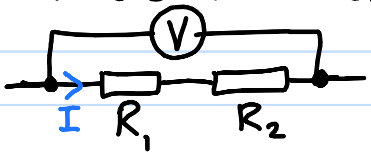
Simplifying circuits can be helpful



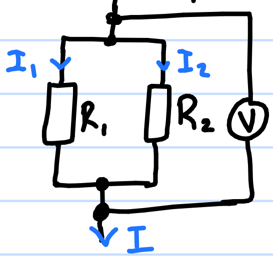
Resistors in series:



KII: $V_1+V_2=V$ use $V=IR$ $\cancel{I}R_1+\cancel{I}R_2=\cancel{I}R_T$

$$R_T = R_1+R_2 \qquad \boxed{R_T=\sum_{n=1}^{N} R_n}$$

Resistors in parallel:

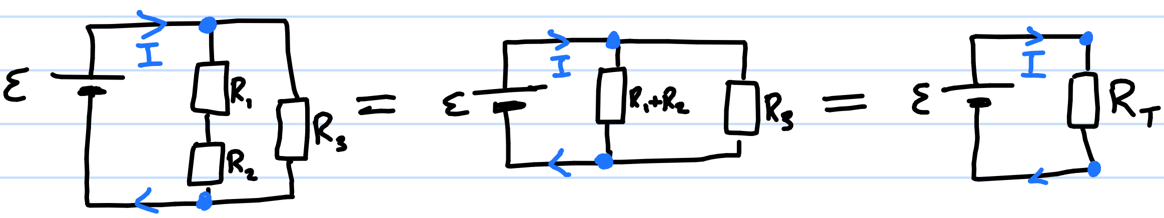


KI: $I_1+I_2=I$ use $V=IR$ $I=\frac{V}{R}$

$$\frac{\cancel{V}}{R_1}+\frac{\cancel{V}}{R_2}=\frac{\cancel{V}}{R_T} \qquad \frac{1}{R_T}=\frac{1}{R_1}+\frac{1}{R_2}$$

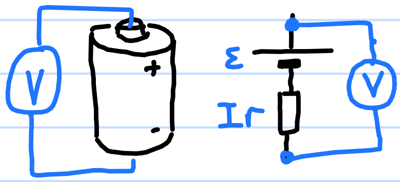
$$\boxed{R_T=\left[\sum_{n=1}^{N}\frac{1}{R_n}\right]^{-1}}$$

Example:

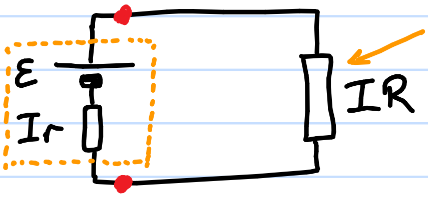


$$\boxed{R_T=\left(\frac{1}{R_1+R_2}+\frac{1}{R_3}\right)^{-1} = \left[(R_1+R_2)^{-1}+R_3^{-1}\right]^{-1}}$$

Electromotive force and internal resistance



V is the terminal p.d.
r is the internal resistance of the supply
KII: $\varepsilon = V + Ir$



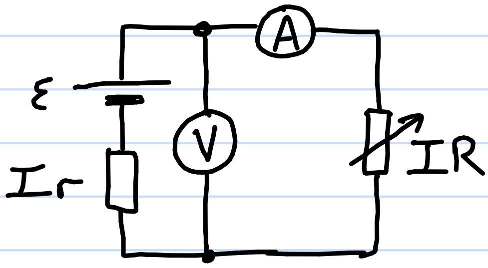
Load resistor
terminal p.d.
Lost volts

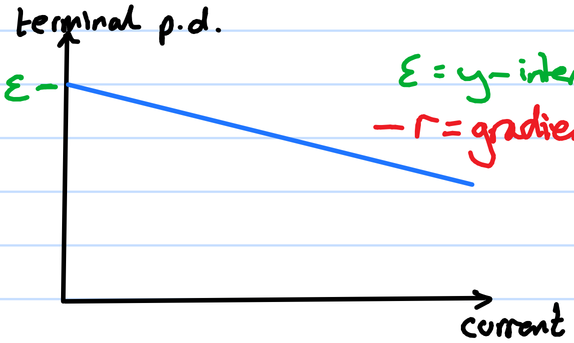
$\varepsilon = IR + Ir$

$V = \varepsilon - Ir \qquad V = IR$

Emf ε will equal terminal p.d when $I = 0$

Measure internal resistance





ε = y-intercept
$-r$ = gradient

$V = \varepsilon - Ir$

terminal pd, emf, lost volts

$$y = m x + c$$
$$V = -r \; I + \varepsilon$$

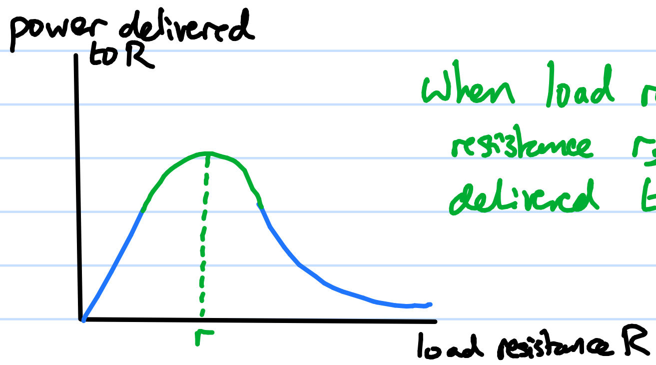
Power

$P = I^2R$
- so does P increase as R increases? If V was constant, yes!
- I decreases as R increases, so does P decrease as R increases?
- We need to include R and r in our calculation of power delivered to load resistor R

$\varepsilon = Ir + IR = I(R+r) \Rightarrow I = \dfrac{\varepsilon}{R+r}$ substitute into $P = I^2R$

$$P = \varepsilon^2 \frac{R}{(R+r)^2}$$

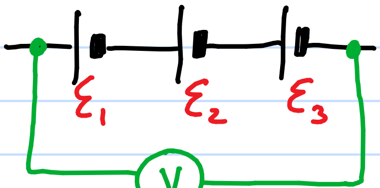
For small R, $P \propto R$
For big R, $P \propto 1/R$



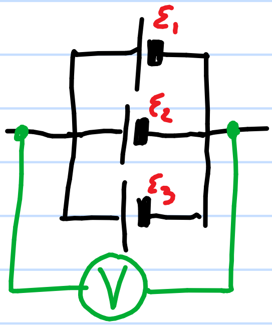
When load resistance R = internal resistance r, maximum power is delivered to R

More circuit calculations

If cells did not have internal resistance?

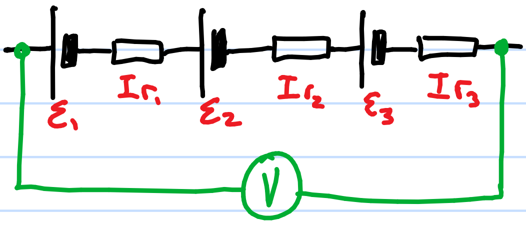


$V = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3$



$V = \mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_3$

If cells do have internal resistance?



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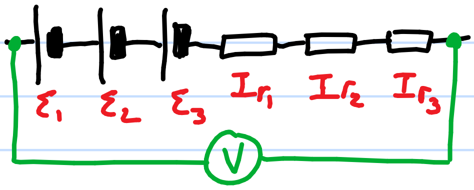
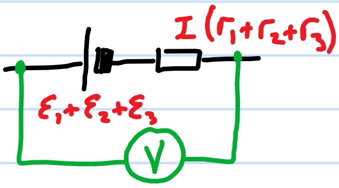


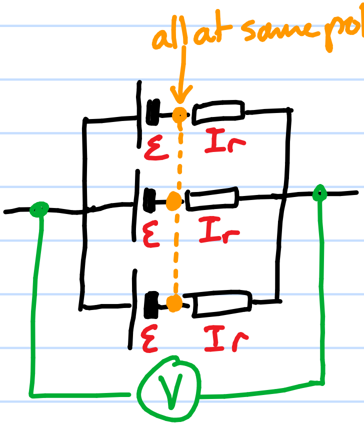
$V = \mathcal{E}_1 - Ir_1 + \mathcal{E}_2 - Ir_2 + \mathcal{E}_3 - Ir_3 \quad = \quad V = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 - Ir_1 - Ir_2 - Ir_3$



In series, we can simply add together emfs, and combine internal resistances into a single resistor.

In parallel

Assume identical cells



If wire connected here, no change to circuit

Combined internal resistance

$r_T = \left[\sum_{n=1}^{N} r_n^{-1}\right]^{-1}$ in this case $r_T = \frac{1}{3} r$

$V = \mathcal{E} - \frac{1}{3} Ir$

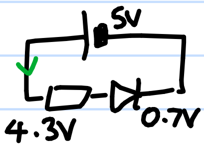
Diodes

Forward bias

Treat diodes like sources of emf in reverse

Drop 0.7V across diode

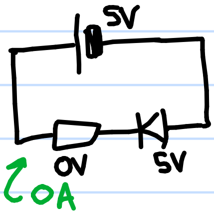
$I = \frac{4.3}{R}$



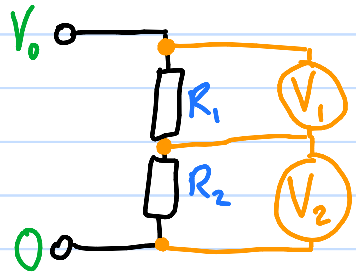
Reverse bias

Treat like open circuit.
i.e. infinite resistance

$V = IR$
$= 0 \times R$



The potential divider



- Because of KI, both resistors have same current passing through through

$$V=IR \qquad V_1=I_1R_1 \qquad V_2=I_2R_2 \qquad V_0=I_0R_0$$

KI : $I_1 = I_2 = I_0$ Combining resistors, $R_0 = R_1 + R_2$
KII : $V_1 + V_2 = V_0$

$$I_0 = I_1 = I_2 \Rightarrow \frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V_0}{R_0}$$

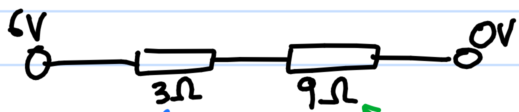
$$\frac{V_1}{R_1} = \frac{V_0}{R_1+R_2} \Rightarrow \frac{V_1}{V_0} = \frac{R_1}{R_1+R_2}$$

$$\frac{V_2}{R_2} = \frac{V_0}{R_1+R_2} \Rightarrow \frac{V_2}{V_0} = \frac{R_2}{R_1+R_2}$$

The share of p.d. across a resistor in a potential divider circuit is equal to its share of the resistance.

$$V_1 = V_0\left(\frac{R_1}{R_1+R_2}\right) \qquad V_2 = V_0\left(\frac{R_2}{R_1+R_2}\right)$$

Example

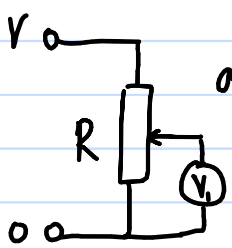


$\frac{1}{4}$ of the resistance so has p.d. of $\frac{1}{4} \times 6V = 1.5V$

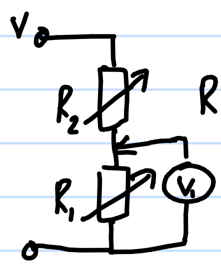
$\frac{3}{4}$ of the resistance so has p.d. of $\frac{3}{4} \times 6V = 4.5V$

$$\boxed{\begin{aligned} I &= \frac{6}{3+9} = 0.50A \\ V &= I\,R \\ V_3 &= 0.50 \times 3 = 1.5V \\ V_9 &= 0.50 \times 9 = 4.5V \end{aligned}}$$

Potentiometer



are just like

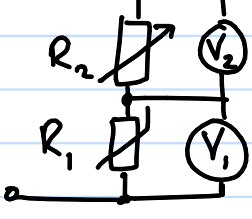


$R_1 + R_2 = R$

Sensing circuits

LDRs, resistance decreases as light intensity increases

Thermistors, resistance decreases as temperature increases



- As temperature increases
- R_1 decreases
- V_1 decrease proportionally to the share of resistance
- V_2 increase!