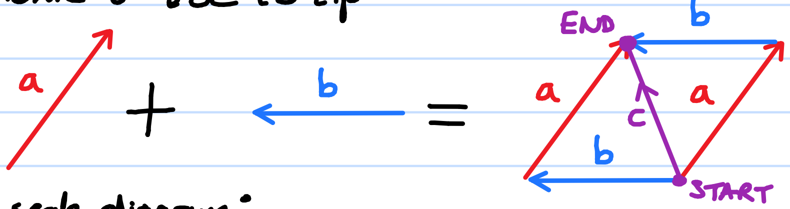
Vectors and scalars

- Vectors have magnitude & direction $\mathbf{x}$, $\vec{x}$, $\underline{x}$, $\underset{\sim}{x}$
- Scalars have magnitude & <u>no</u> direction x

Scalar	Vector
distance	displacement
speed	velocity
energy	Force
	acceleration

Addition of vectors (graphical):

Mnemonic: "toe to tip"

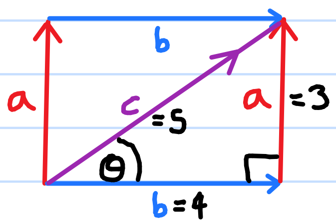


using scale diagram:

- Choose scale
- Vectors represented as displacements

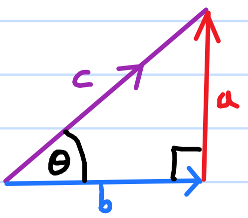
- $\vec{a}$ & $\vec{b}$ are components
- $\vec{c}$ is resultant

Algebraically adding two perpendicular vectors:



$a = 3m$
$b = 4m$
$c = \sqrt{a^2 + b^2}$ (Pythagorean)
$\theta = \arctan\left(\frac{a}{b}\right) = \arctan\left(\frac{3}{4}\right) = 36.9°$
$\vec{a} + \vec{b} = 5m \measuredangle 36.9°$ acw from $\vec{b}$

Resolving vectors into perpendicular components:



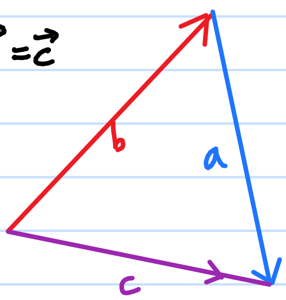
$\vec{a} = c \sin\theta \ \hat{j}$
$\vec{b} = c \cos\theta \ \hat{i}$

$\vec{c} = a\hat{j} + b\hat{i}$

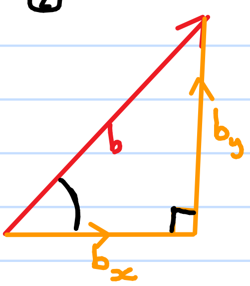
unit vectors:
Direction ✓
Dimension ✗
Units ✗
Magnitude = 1

Algebraic addition of two non-perpendicular vectors:

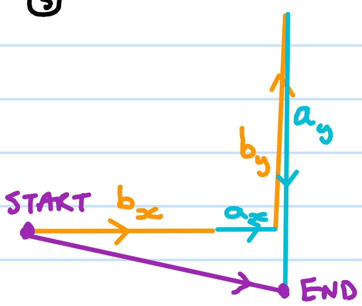
① $\vec{a} + \vec{b} = \vec{c}$



②



③



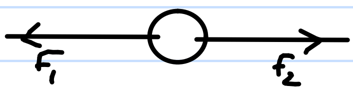
$c_x = b_x + a_x$
$c_y = a_y - b_y$

$$\vec{c} = a_x\hat{i} + b_x\hat{i} + a_y\hat{j} - b_y\hat{j} = (a_x + b_x)\hat{i} + (a_y - b_y)\hat{j}$$

Balanced Forces

- For a body to be in equilibrium, forces must be balanced (and moments must be)
- No acceleration of a body in equilibrium
- Resultant force in equilibrium is zero

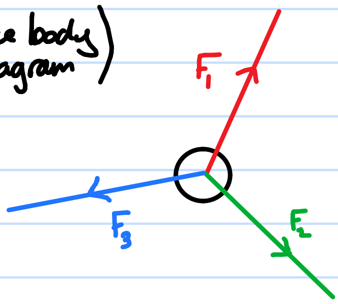
Two forces in balance:



$\vec{F_1} = -\vec{F_2}$ $\therefore$ $\vec{F_1} + \vec{F_2} = 0$

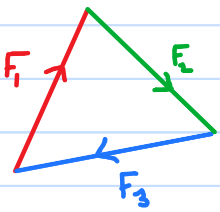
Three forces in balance:

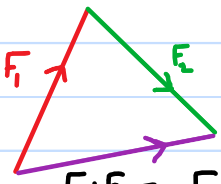
(Free body diagram)



=

(Force vector diagram)

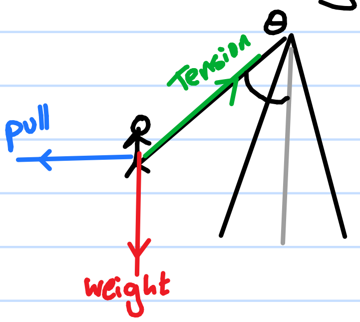


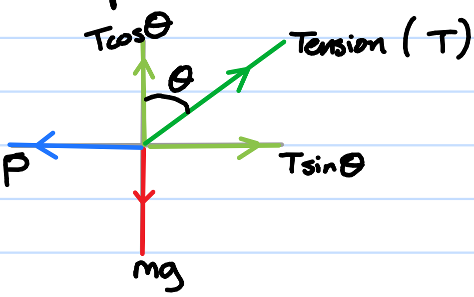


$F_1 + F_2 = -F_3$

What force is needed to balance F_1 & F_2?

Child on swing example:



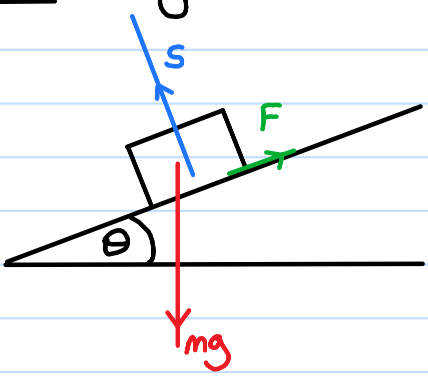


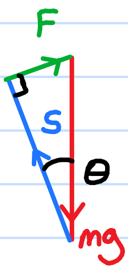
$$\left.\begin{aligned} P &= T\sin\theta \\ mg &= T\cos\theta \end{aligned}\right\} \text{simultaneous equations}$$

$$\frac{P}{mg} = \frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \tan\theta$$

$$P = mg\tan\theta$$

Inclines (rough)



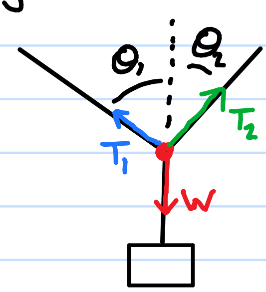


$$F = mg\sin\theta$$
$$S = mg\cos\theta$$

Resultant force if no friction $= mg\sin\theta$

$$\cancel{m}a = \cancel{m}g\sin\theta$$
$$a = g\sin\theta$$

Suspending weight with two cords



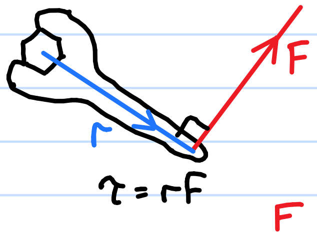
$T_1\sin\theta_1 = T_2\sin\theta_2$ ← horizontal

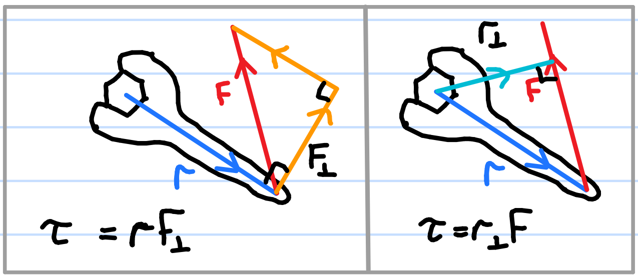
$W = T_1\cos\theta_1 + T_2\cos\theta_2$ ← vertical

The principle of moments (moment, AKA torque)

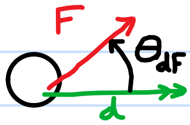
Definition of moment (NOT principle of moments)

$\vec{\tau} = \vec{r} \times \vec{F}$



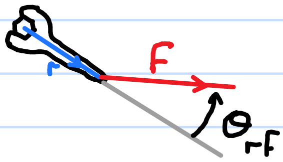


- Scalar product : $W = \vec{d} \cdot \vec{F}$



$W = \vec{d} \cdot \vec{F} = |d||F| \cos \theta_{dF}$

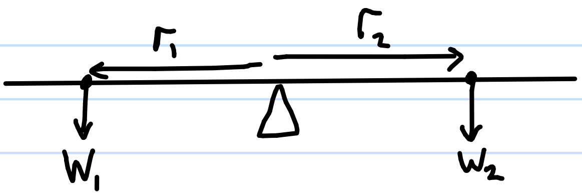
- Vector product : $\vec{\tau} = \vec{r} \times \vec{F}$ $\quad \vec{\tau} = |r||F| \sin \theta_{rF} \, \hat{\tau}$



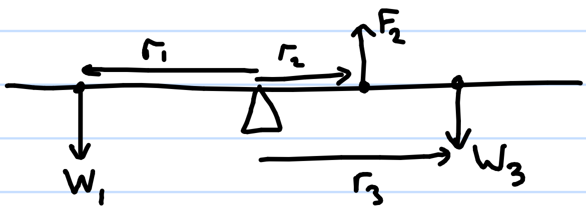
τ is turning effect of a force

The principle of Moments

- For a body in equilibrium,
- the sum of clockwise moments = the sum of anticlockwise moments.



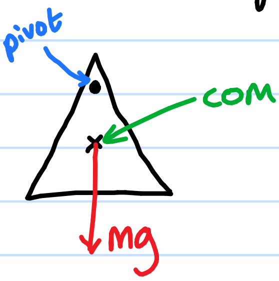
$r_1 W_1 = r_2 W_2$



$r_1 W_1 + r_2 F_2 = r_3 W_3$

Centre of Mass (com)

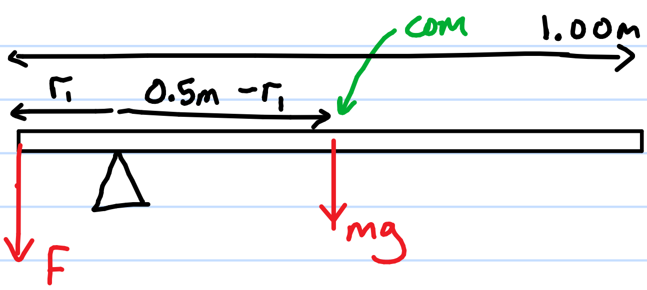
- Where the weight is considered to act



- body will settle with com beneath pivot

$$M = \frac{r_1 F}{(0.5 - r_1) g}$$

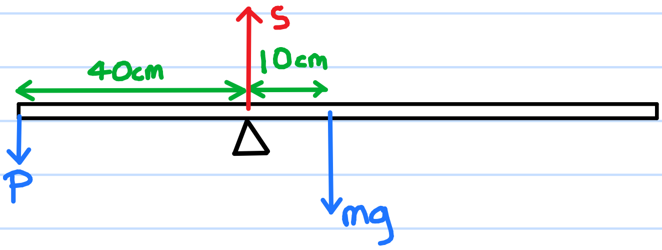
Weighing a metre ruler



- Measure r_1 & F
- $r_1 F = (0.5 - r_1) mg$ $\quad$ solve for M

More on moments

Single support problems:



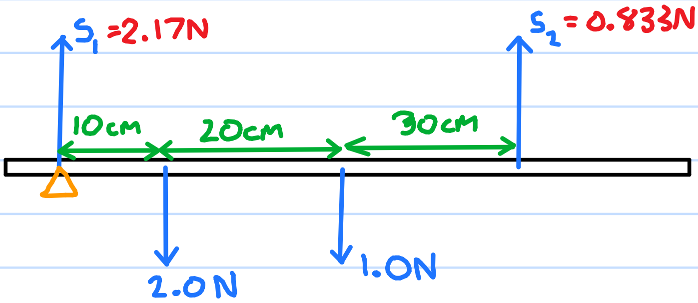
$m = 100\,g \quad g = 10\,N\,kg^{-1} \quad mg = 1.0\,N$

$\tau_{cw} = 10 \times 0.100 \times 10 = 10\,Ncm$
$= \tau_{acw} = 10\,Ncm = 40\,cm \times P$
$P = 0.25\,N$

Total Force down $= 1.0 + 0.25$
$= 1.25\,N$
$\therefore S = -1.25\,N$

Two support problems

We can arbitrarily choose the pivot, we can simplify calculations by judicious choice of pivot location

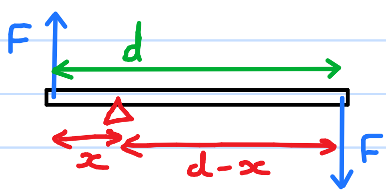


Because translational equilibrium; $S_1 + S_2 = 3.0\,N$

$\tau_{cw} = 10 \times 2.0 + 30 \times 1.0 = 50\,Ncm$
$\tau_{acw} = 50\,Ncm = 60 \times S_2$
$S_2 = \frac{5}{6}\,N = 0.833\,N$

$S_1 = 3.0 - 0.833 = 2.17\,N$

Couples:



Couples = two forces of same magnitude, opposite direction, not along same line of action

d = perpendicular displacement between lines of action of two antiparallel forces

$$\tau = xF + (d-x)F = \cancel{xF} + dF - \cancel{xF}$$
$$= dF$$

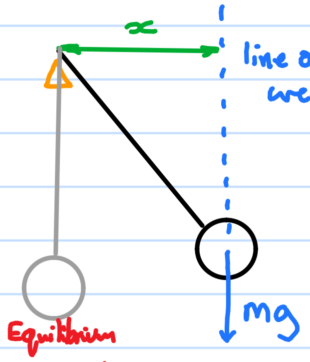
i.e. moment of the couple

Stability

Stable equilibrium : perturbed body resultant moment acts to restore equilibrium
Unstable equilibrium : perturbed body resultant moment acts to increase resultant moment
Neutral equilibrium : perturbed body resultant moment is zero

STABLE

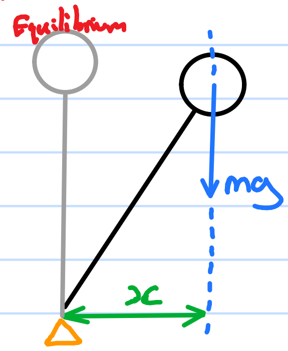
when m directly beneath pivot, stable equilibrium



resultant moment xmg is clockwise
as mass falls, x decreases, so resultant moment decreases

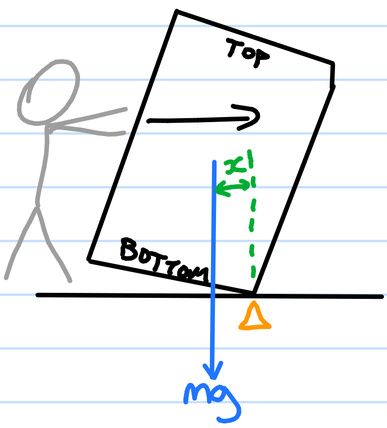
UNSTABLE

when m directly over pivot, unstable equilibrium



resultant moment xmg is clockwise
as mass falls, x increases, so resultant moment increases

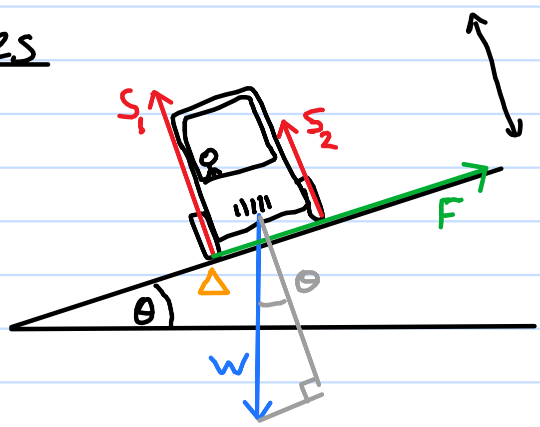
Tilting vs toppling



when tilting, xmg is acw, so when released, wardrobe returns to upright position

Toppling is when centre of mass passes pivot so resultant moment becomes clockwise as drawn in this illustration

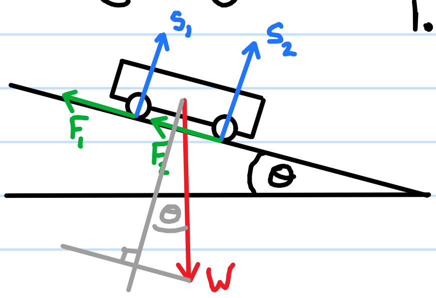
Slopes



parallel to slope : $F = W\sin\theta$
perpendicular to slope : $S_1 + S_2 = W\cos\theta$

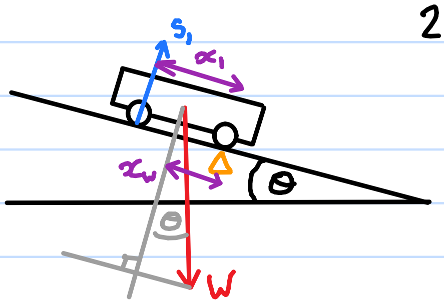
Equilibrium rules

Free body diagrams



1. In equilibrium, vector sum of W, S_1, S_2, F_1, and $F_2 = 0$
   Force vector diagram shows a closed shape
   Break vectors down to perpendicular components, equate to zero seperately

$$S_1 + S_2 = W\cos\theta \quad \& \quad F_1 + F_2 = W\sin\theta$$



2. Principle of moments applies
   Not drawn forces that do not produce moment around pivot

$$x_1 S_1 = x_w W\cos\theta$$

If pivoted at other wheel

$$x_1 S_2 = (x_1 - x_w) W\cos\theta$$

Here are <u>four</u> simultaneous equations to solve

Quantities: S_1, S_2, F_1, F_2, W, x_1, x_w, θ → if any <u>four</u> are given, the other <u>four</u> can be calculated using

$$S_1 + S_2 = W\cos\theta$$
$$F_1 + F_2 = W\sin\theta$$
$$x_1 S_1 = x_w W\cos\theta$$
$$x_1 S_2 = (x_1 - x_w) W\cos\theta$$

Statics Calculations

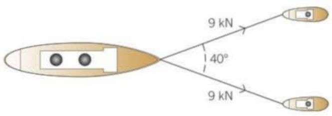
1 Calculate the magnitude of the resultant of a 6.0 N force and a 9.0 N force acting on a point object when the two forces act:
  a in the same direction
  b in opposite directions
  c at 90° to each other.

2 A point object in equilibrium is acted on by a 3 N force, a 6 N force, and a 7 N force. What is the resultant force on the object if the 7 N force is removed?

3 A point object of weight 5.4 N in equilibrium is acted on by a horizontal force of 4.2 N and a second force *F*.
  a Draw a free body force diagram for the object and determine the magnitude of *F*.
  b Calculate the angle between the direction of *F* and the horizontal.

4 An object of weight 7.5 N hangs on the end of a cord, which is attached to the midpoint of a wire stretched between two points on the same horizontal level, as shown in Figure 1. Each half of the wire is at 12° to the horizontal. Calculate the tension in each half of the wire.

5 A ship is towed at constant speed by two tugboats, each pulling the ship with a force of 9.0 kN. The angle between the tugboat cables is 40°, as shown in Figure 2.



▲ Figure 2

  a Calculate the resultant force on the ship due to the two cables.
  b Calculate the drag force on the ship.

6 A metre rule of weight 1.0 N is pivoted on a knife-edge at its centre of mass, supporting a weight of 5.0 N and an unknown weight *W* as shown in Figure 3. To balance the rule horizontally with the unknown weight on the 250 mm mark of the rule, the position of the 5.0 N weight needs to be at the 810 mm mark.
  a Calculate the unknown weight.
  b Calculate the support force on the rule from the knife-edge.

7 In Figure 3, a 2.5 N weight is also suspended from the rule at its 400 mm mark. What adjustment needs to be made to the position of the 5.0 N weight to rebalance the rule?

8 A uniform metre rule is balanced horizontally on a knife-edge at its 350 mm mark, by placing a 3.0 N weight on the rule at its 10 mm mark.
  a Sketch the arrangement and calculate the weight of the rule.
  b Calculate the support force on the rule from the knife-edge.

9 A uniform diving board has a length 4.0 m and a weight of 250 N, as shown in Figure 4. It is bolted to the ground at one end and projects by a length of 3.0 m beyond the edge of the swimming pool. A person of weight 650 N stands on the free end of the diving board. Calculate:
  a the force on the bolts
  b the force on the edge of the swimming pool.

10 A uniform beam XY of weight 1200 N and length 5.0 m is supported horizontally on a concrete pillar at each end. A person of weight 500 N sits on the beam at a distance of 1.5 m from end X.
  a Sketch a free body force diagram of the beam.
  b Calculate the support force on the beam from each pillar.

11 A bridge crane used at a freight depot consists of a horizontal span of length 12 m fixed at each end to a vertical pillar, as shown in Figure 5.
  a When the bridge crane supports a load of 380 kN at its centre, a force of 1600 kN is exerted on each pillar. Calculate the weight of the horizontal span.
  b The same load is moved across a distance of 2.0 m by the bridge crane. Sketch a free body force diagram of the horizontal span and calculate the force exerted on each pillar.

12 A uniform curtain pole of weight 24 N and of length 3.2 m is supported horizontally by two wall-mounted supports X and Y, which are 0.8 m and 1.2 m from each end, respectively.
  a Sketch the free body force diagram for this arrangement and calculate the force on each support when there are no curtains on the pole.
  b When the pole supports a pair of curtains of total weight 90 N drawn along the full length of the pole, what will be the force on each support?

13 A uniform steel girder of weight 22 kN and of length 14 m is lifted off the ground at one end by means of a crane. When the raised end is 2.0 m above the ground, the cable is vertical.
  a Sketch a free body force diagram of the girder in this position.
  b Calculate the tension in the cable at this position and the force of the girder on the ground.

14 A rectangular picture 0.80 m deep and 1.0 m wide, of weight 24 N, hangs on a wall, supported by a cord attached to the frame at each of the top corners, as shown in Figure 6. Each section of the cord makes an angle of 25° with the picture, which is horizontal along its width.
  a Copy the diagram and mark the forces acting on the picture on your diagram.
  b Calculate the tension in each section of the cord.

1. a) $6.0+9.0=15N$

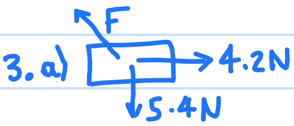
b) $9.0-6.0=3.0N$

c) $\sqrt{6.0^2+9.0^2}=108N$

$\approx 110N$

2. 7N

3. a)



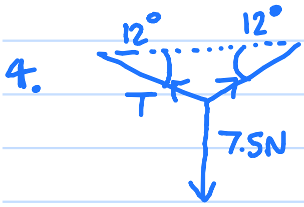
$F=\sqrt{4.2^2+5.4^2}=6.84N$

$\approx 6.8N$

b) $\theta = \tan^{-1}(5.4 \div 4.2)$

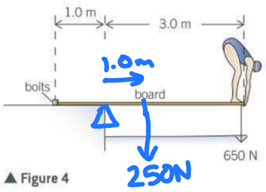
$=52°$

4.

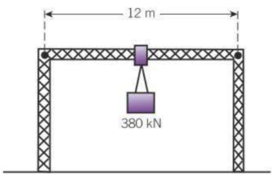


$\frac{1}{2}\times 7.5 = T\sin 12$

$T=18N$



▲ Figure 4



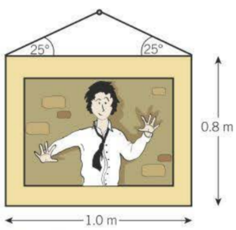
▲ Figure 5

9. a) $\frac{1.0\times 250+3.0\times 650}{1.0}$

$=2200N$

b) $2200+250+650$

$=3100N$



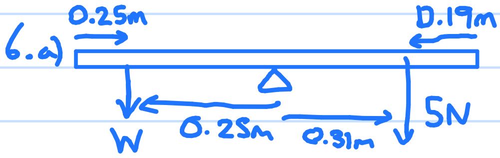
▲ Figure 6

5. a) $2\times 9\times 10^3\times\cos 20 = 16.9kN$

$\approx 17kN$

b) 16.9kN

6. a)

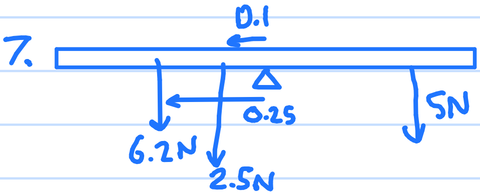


$0.25\times W = 0.31\times 5$

$W=\frac{0.31}{0.25}\times 5 = 6.2N$

b) $5.0+6.2+1.0=12.2N$

7.



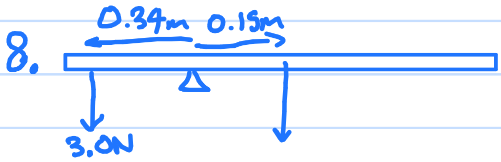
$\tau_{acw}=1.8Nm$

5N must be at $0.5+0.36=$

0.86m from left of ruler

So move 5cm to right

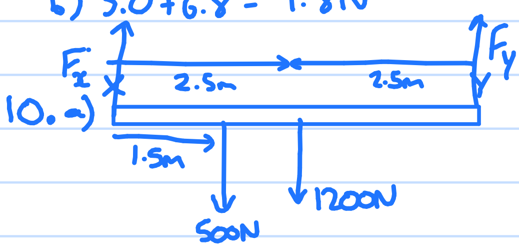
8.



a) $\frac{0.34}{0.15}\times 3.0 = 6.8N$

b) $3.0+6.8=9.8N$

10. a)



b) x as pivot: $1.5\times 500+2.5\times 1200$

$=5.0\times F_y$

$F_y=750N$

$F_x=500+1200-750$

$=950N$