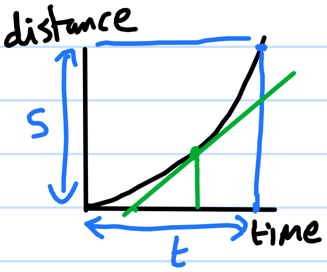
Speed and velocity

speed = $\frac{\text{distance}}{\text{duration}}$ good enough for GCSE

average speed = $\frac{\text{total distance}}{\text{total duration}}$



$v_{av} = \frac{s}{t}$

$v = \frac{ds}{dt}$ = gradient of tangent = instantaneous speed

gradient of straight line $= \frac{\Delta s}{\Delta t}$

gradient of curve $= \frac{ds}{dt}$

For constant speed, distance – time graph straight line

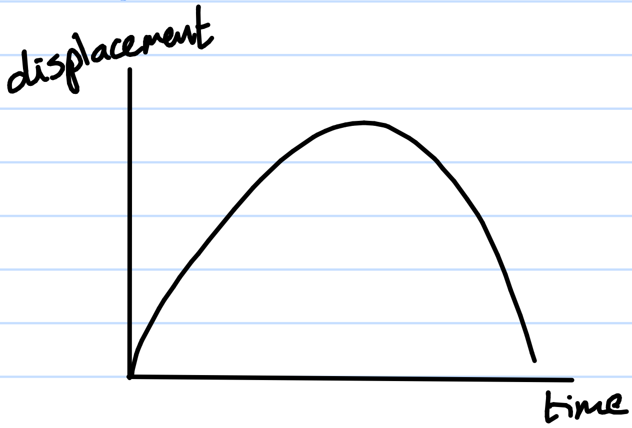
For motion in a circle of constant speed $v = \frac{2\pi r}{T} = 2\pi f r$

For constant speed, instantaneous speed = average speed

Velocity: $\vec{v} = \frac{\vec{s}}{t}$ or $\vec{s} = \vec{v}t$ (average velocity) $\vec{s}$ = displacement

distance cannot reduce, distance is space through which a body moves

displacement can reduce, shortest line connecting the start of motion of a body to the end



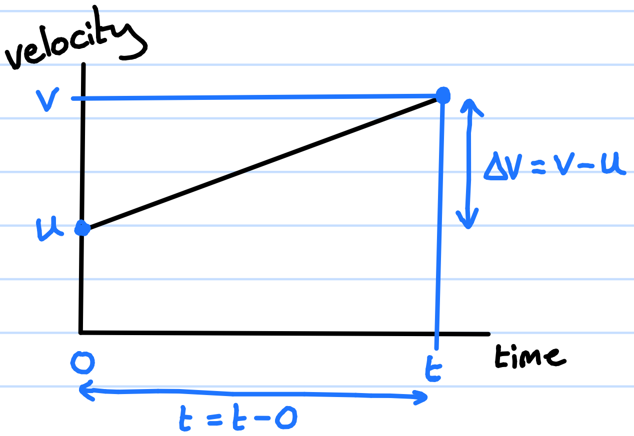
$\vec{v} = \frac{d}{dt}\vec{s}$ (graphical determination by drawing a tangent)

Acceleration

Acceleration is rate of change of velocity.

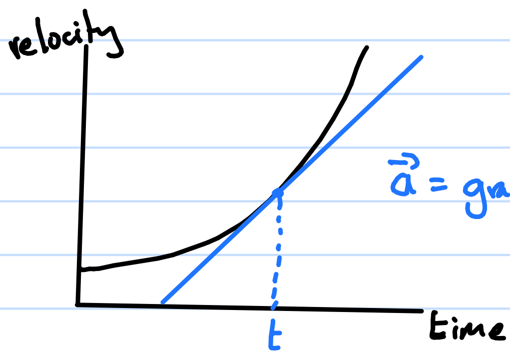
$\vec{a} = \frac{d}{dt}\vec{v}$ or for uniform acceleration $\vec{a} = \frac{\Delta\vec{v}}{t}$ $\quad \Delta\vec{v} = \vec{v} - \vec{u}$

$\vec{v}$ = Final velocity $\quad \vec{u}$ = initial velocity $\quad \vec{a} = \frac{\vec{v}-\vec{u}}{t}$ or $\boxed{\vec{v} = \vec{u} + \vec{a}t}$



$\vec{a} = \frac{\vec{v}-\vec{u}}{t}$ is the gradient

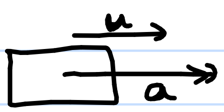
For non-uniform acceleration:



$\vec{a}$ = gradient of tangent

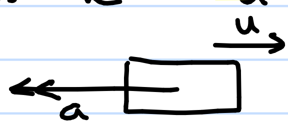
Acceleration or deceleration?

1. u is +ve $\quad$ a is +ve



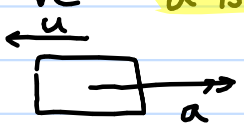
speed will increase

2. u is +ve $\quad$ a is −ve



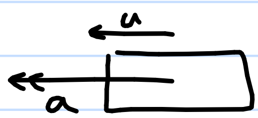
speed decrease ∴ decelerating

3. u is −ve $\quad$ a is +ve



speed will decrease ∴ decelerating

4. u is −ve $\quad$ a is −ve



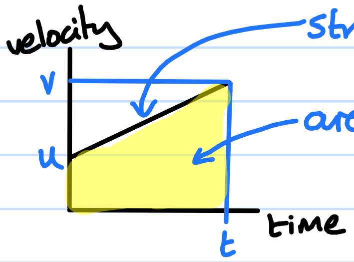
speed will increase

deceleration is the rate of reduction of speed.
" is a scalar!

Motion along a straight line at constant acceleration

s = displacement u = initial velocity v = final velocity a = acceleration
t = duration of motion

① no a: definition of average velocity $v_{av} = \frac{s}{t}$ $s = v_{av} t$



s = area of trapezium $= \left(\frac{v+u}{2}\right)t$

$v_{av} = \frac{v+u}{2}$ only works for constant a

$\frac{v+u}{2} = \frac{s}{t} \Rightarrow \boxed{s = \frac{(v+u)t}{2}}$

② no s: definition of acceleration $a = \frac{v-u}{t}$

$\boxed{v = u + at}$ only valid for constant a

③ no t: combine ① and ②

①$\Rightarrow t = \frac{2s}{v+u}$ ②$\Rightarrow t = \frac{v-u}{a}$

$$\frac{2s}{v+u} = \frac{v-u}{a}$$

$$2as = (v+u)(v-u) = v^2 \cancel{+uv} \cancel{-uv} - u^2$$
$$= v^2 - u^2$$
$$\boxed{v^2 = u^2 + 2as}$$

④ no v:
①$\Rightarrow v = \frac{2s}{t} - u$ ②$\Rightarrow v = u + at$

$$\frac{2s}{t} - u = u + at$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

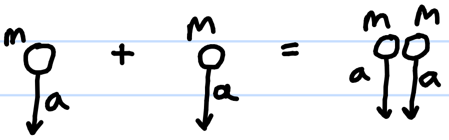
⑤ no u:
①$\Rightarrow u = \frac{2s}{t} - v$ ②$u = v - at$

$$\frac{2s}{t} - v = v - at$$

$$\boxed{s = vt - \frac{1}{2}at^2}$$

Freefall

- All objects in freefall have same acceleration (no air resistance)



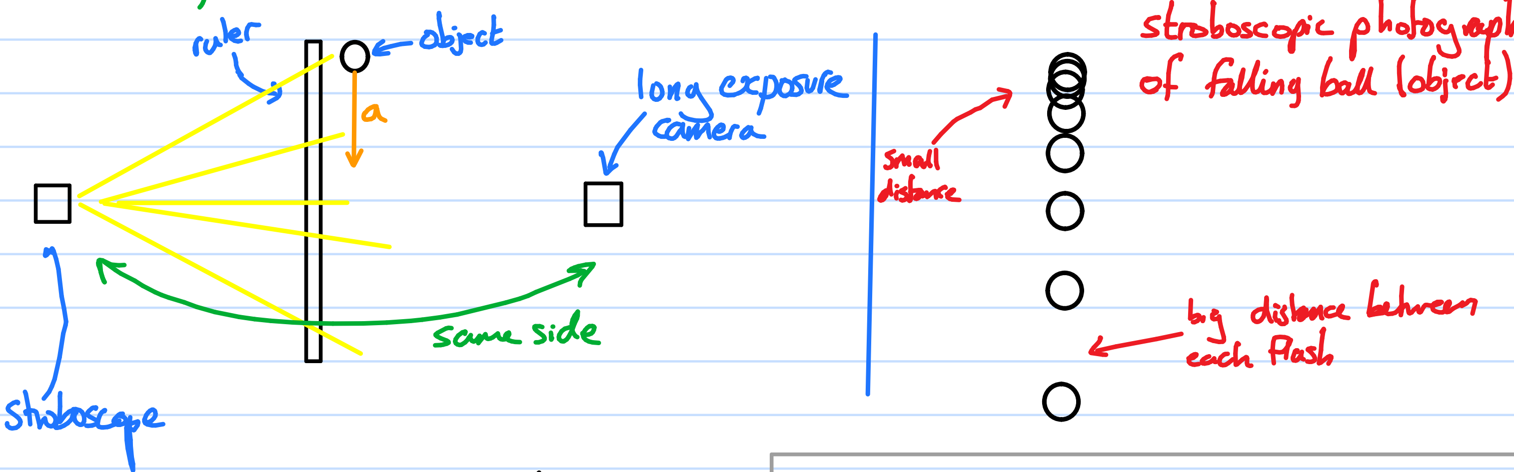
(Galileo's thought experiment)

- Using inclined plane and water-drip clock, Galileo investigated acceleration

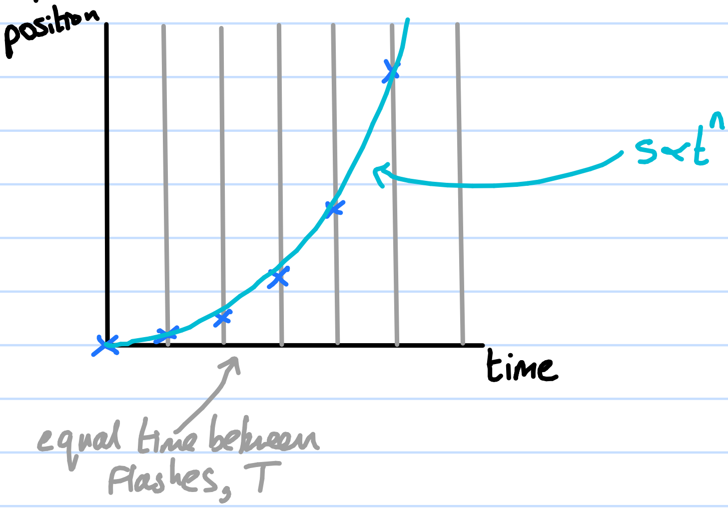
- "modern" techique for investigating freefall: stroboscopy

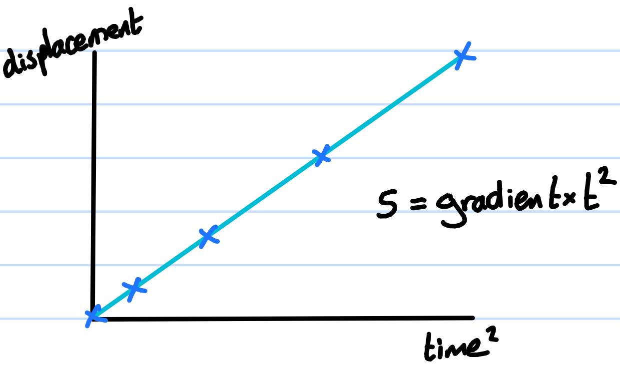
a stroboscope flashes at regular intervals, can measure frequency of flashes

$T = \frac{1}{f}$, T time between each flash



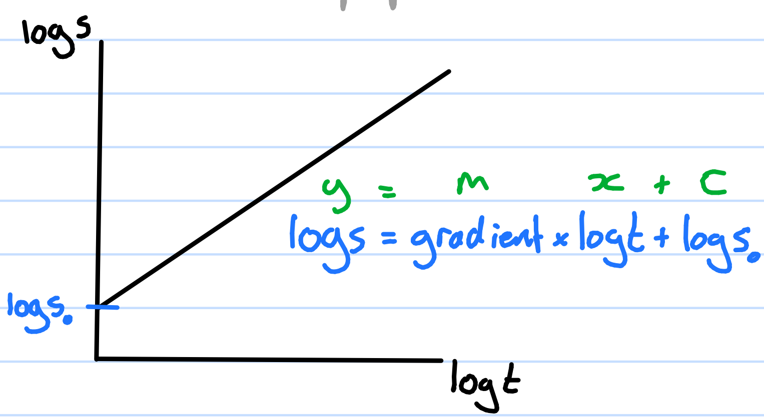
plot ball position – time graph:





$\boxed{g = 2 \times \text{gradient}}$

Power law (paper 3A AQA)



in this experiment, gradient = 2

$$\log s = 2 \times \log t + \log s_0$$
$$= \log t^2 + \log s_0$$
$$s = s_0 t^2 \qquad s \propto t^2$$

s = y-axis
u = 0
v = X
a = ?
$t = \sqrt{x\text{-axis}}$

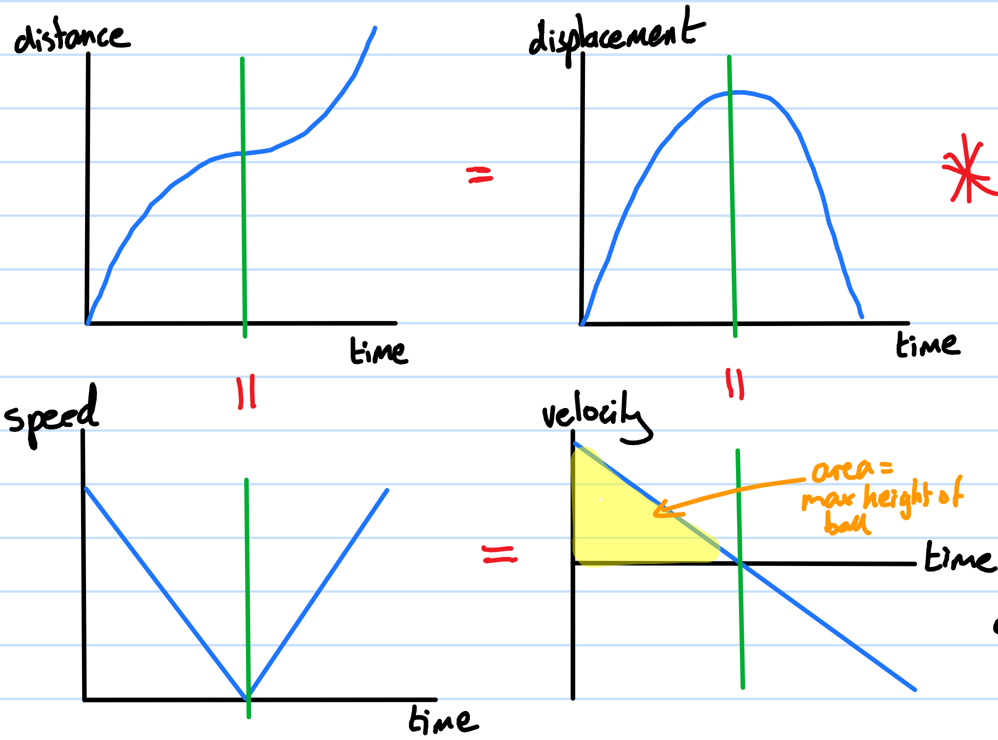
$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}at^2$

$\text{gradient} = \frac{1}{2}a = \frac{1}{2}g$

acceleration due to gravity

Motion graphs



- For both, higher magnitude gradient means higher speed
- Magnitude of gradient = speed
- Graphs show a ball thrown in the air
- Gradient displacement-time graph is velocity
- For velocity-time graphs, gradient is acceleration
- for speed-time, only reduction in speed is interesting |gradient| = deceleration

- Area under is distance
- scalar!

- Area under (between line and x-axis) = s
- Area = s is +ve if above x-axis, is -ve if below

More calculations on motion along a straight line

$v = u + at$ ① no s
$s = \frac{1}{2}(u+v)t$ ② no a
$s = ut + \frac{1}{2}at^2$ ③ no v
$v^2 = u^2 + 2as$ ④ no t

$\left.\right\}$ Only valid for constant acceleration

Example 1) car moving at $2.5 ms^{-1}$, 26m from garage. Decelerates uniformly to come to rest at the garage.
a) deceleration? b) time to stop?

a) $s = 26m$
$u = 2.5 ms^{-1}$
$v = 0 ms^{-1}$
$a = ?$
$t = X$

use no t equation $v^2 = u^2 + 2as$

$$a = \frac{v^2 - u^2}{2s} = \frac{-2.5^2}{2 \times 26} = \frac{-6.25}{52} = -0.120 \, ms^{-2}$$

deceleration $= 0.12 \, ms^{-2}$

b) $s = 26m$
$u = 2.5 ms^{-1}$
$v = 0 ms^{-1}$
$a = X$
$t = ?$

use no a equation $s = \frac{1}{2}(u+v)t$

$$t = \frac{2s}{u+v} = \frac{2 \times 26}{2.5 - 0} = 20.8$$
$\approx 21 \, s$

Example 2) stage 1, cup in freefall
stage 2, cup crumpling when colliding with floor

Final velocity v of stage 1 is initial velocity u of stage 2

Stage 1) $s = -0.85m$
$u = 0 ms^{-1}$
$v = ?$
$a = -9.81 \, ms^{-2}$
$t = X$

use no t equation $v^2 = u^2 + 2as$

$$v = \pm\sqrt{u^2 + 2as} = \pm\sqrt{0^2 + 2 \times (-9.81) \times (-0.85)}$$
$= \pm 4.08 \, ms^{-1}$
$\approx -4.1 \, ms^{-1}$

stage 2) $s = -0.025m$
$u = -4.08 \, ms^{-1}$
$v = 0 ms^{-1}$
$a = ?$
$t = X$

use no t equation $v^2 = u^2 + 2as$

$$a = \frac{v^2 - u^2}{2s} = \frac{0^2 - (-4.08)^2}{2 \times (-0.025)} = \frac{-16.65}{-0.050}$$

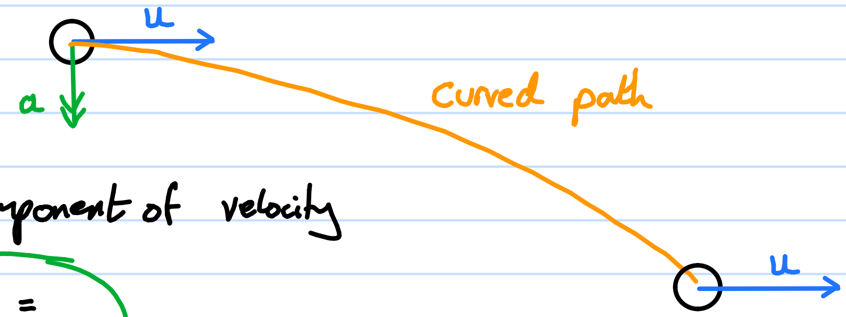
$= 333 \, ms^{-2}$
$= 3.3 \times 10^2 \, ms^{-2}$ if unrounded used
$= 336 \, ms^{-2}$ if $-4.1 \, ms^{-1}$ used
$= 3.4 \times 10^2 \, ms^{-2}$

Projectile motion 1 — assumes acceleration is constant

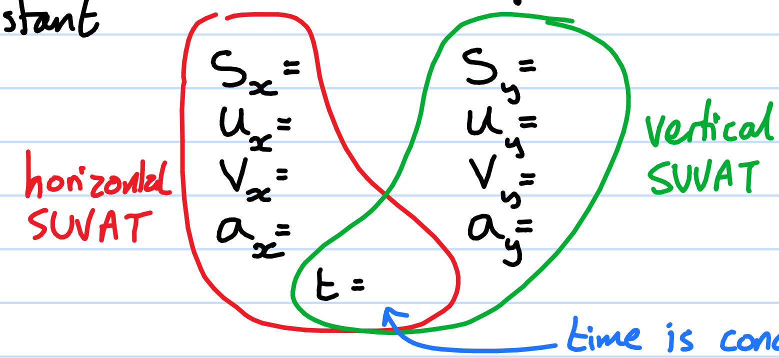
- If initial velocity of body is horizontal, we call it "horizontal projection"
- " vertical, " "vertical projection"
- " diagonal, " "diagonal projection"

- We have already looked at vertical projection last lesson. — i.e. 1-dimensional projection

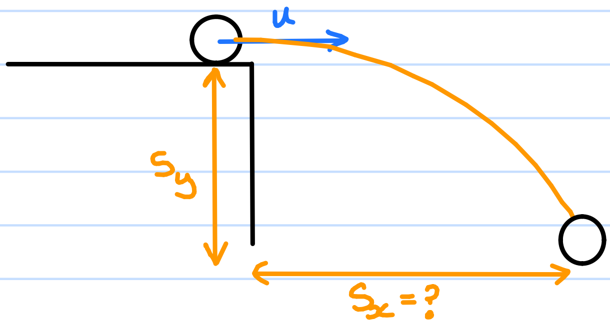
Horizontal projection:



- a is vertical, so horizontal component of velocity is constant



Example: ball rolling off desk, 0.70 m high desk, ball rolls at 0.90 ms^{-1}



$s_y = -0.70\,m$	$s_x = ?$
$u_y = 0\,ms^{-1}$	$u_x = 0.90\,ms^{-1}$
$v_y =$	$v_x =$
$a_y = -9.81\,ms^{-2}$	$a_x = 0\,ms^{-2}$

$t = 0.378\,s$

- Step 1 - calculate t from vertical motion
- Step 2 - calculate s_x from horizontal and t

- $v_x^2 = u_x^2 + 2a_x s_x \Rightarrow v_x = u_x$ cannot use to calculate s_x

- Use vertical motion to calculate t
- no v_y equation $s_y = u_y t + \frac{1}{2}a_y t^2$

$$t = \pm\sqrt{\frac{2s_y}{a_y}} = \sqrt{\frac{2\times(-0.70)}{-9.81}} = 0.378\,s$$

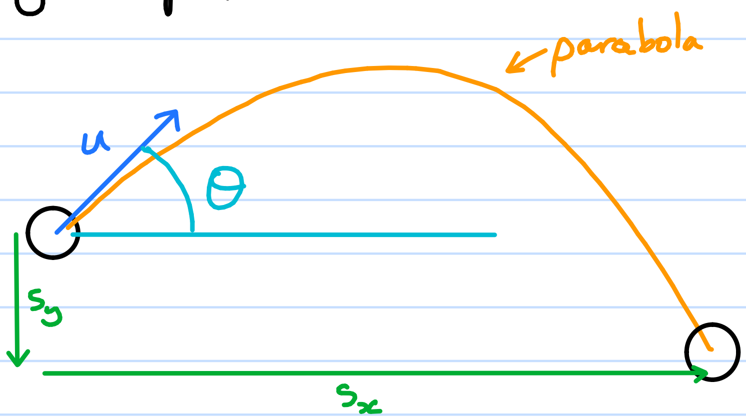
- Use no v_x equation $s_x = u_x t + \frac{1}{2}a_x t^2$, $a_x = 0$

$s_x = u_x t = 0.90 \times 0.378 = 0.340\,m$

$\approx 3.4 \times 10^{-1}\,m$

Projectile motion 2

Diagonal projectile motion:



$u_x = u\cos\theta$ $\quad u_y = u\sin\theta$

$u = 10\,ms^{-1}$ $\quad \theta = 30°$
$s_y = -3.5m$ $\quad a_y = -9.81\,ms^{-2}$
$s_x = ?$ $\quad a_x = 0\,ms^{-2}$

$s_y = -3.5m$	$s_x = ?$
$u_y = 10\sin 30 = 5\,ms^{-1}$	$u_x = 10\cos 30 = 8.66\,ms^{-1}$
$v_y = -9.00\,ms^{-1}$	$v_x =$
$a_y = -9.81\,ms^{-2}$	$a_x = 0\,ms^{-2}$

$t = 1.43s$

no v_y : $s_y = u_y t + \frac{1}{2}a_y t^2$ ✗
no t : $v_y^2 = u_y^2 + 2a_y s_y$

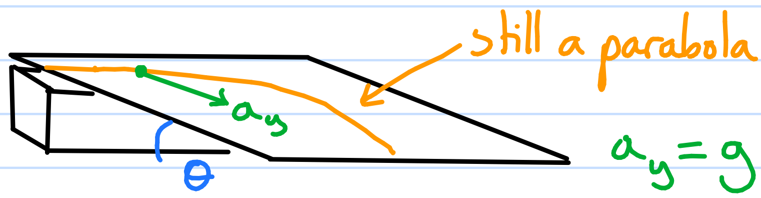
$$v_y = \pm\sqrt{(-3.5)^2 + 2\times(-9.81)(-3.5)}$$
$$= -9.00\,ms^{-1}$$

no s_y : $v_y = u_y + a_y t$

$$t = \frac{v_y - u_y}{a_y} = \frac{-9.00-5}{-9.81} = 1.43s$$

no v_x : $s_x = u_x t + \frac{1}{2}a_x t^2 = u_x t = 8.66\times1.43$
$= 12.4m$

Projections on inclines:



$a_y = g\sin\theta$

resolve acceleration onto the plane

Projectile-like motion:

- falling under gravity
- charges in uniform electric fields
- NOT charges in uniform magnetic fields
- NOT orbital mechanics

Air resistance?

- smaller displacement
- lower max height
- reach max height sooner