Momentum and impulse

momentum, $p = mv$

$p = \frac{h}{\lambda}$ non-classical

Newton's 1st & 2nd law:

1. if $\Sigma F = 0$, $\frac{d}{dt}p = 0$

$\frac{d}{dt}$ = rate of change

for discrete changes $\frac{\Delta p}{t} = 0$

2. $\Sigma F \propto \frac{d}{dt}p$

constant of proportionality = 1 $\therefore$ $\Sigma F = \frac{dp}{dt} = \frac{d}{dt}p$

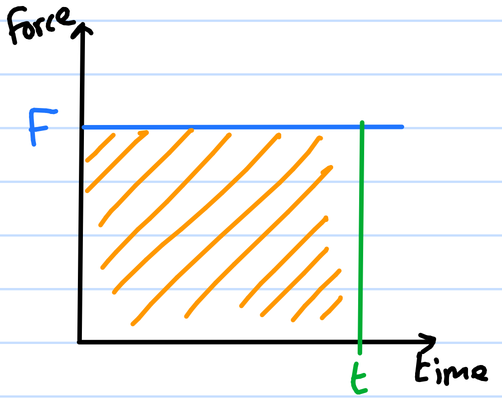
what if $\frac{dm}{dt} = 0$? (ie. const mass) $F = \frac{d(mv)}{dt} = m\frac{dv}{dt} + \underbrace{v\frac{dm}{dt}}_{0} = m\frac{dv}{dt} = ma$

what if $\frac{dm}{dt} \neq 0$? (i.e. changing mass) $F = m\frac{dv}{dt} + v\frac{dm}{dt} = \underbrace{ma}_{} + \underbrace{v(t)\frac{dm}{dt}}_{}$

$\boxed{\text{Impulse is change in momentum}}$

$F = \frac{dp}{dt} \rightarrow \int F \cdot dt = \int dp$

if $\frac{dF}{dt} = 0$, $F\int dt = \int dp \rightarrow Ft = \Delta p$



$Ft = \Delta p$ = area of graph between line and x-axis

Area of F-t graph is equal to impulse

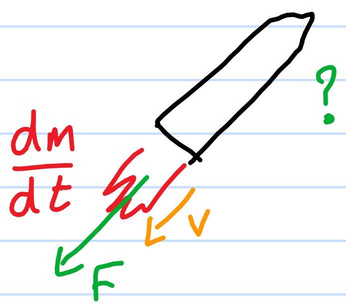
$F = \frac{\Delta p}{t} = \frac{mv - mu}{t} = \frac{m(v-u)}{t} \Rightarrow v - u = \frac{Ft}{m}$

Rocket launch:

- Assume const v rocket launch $a = 0$
- Mass of rocket reducing because of using fuel propellant

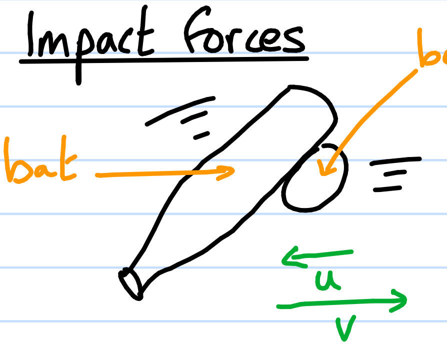
$F = v\frac{dm}{dt} + \underbrace{m\frac{dv}{dt}}_{=0}$

$F = v\frac{dm}{dt}$ Force due to expulsion of mass at particular velocity

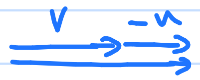


Newton's third law!

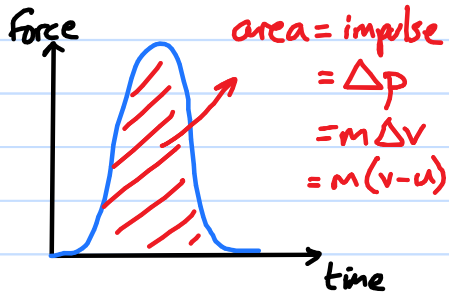
Impact Forces



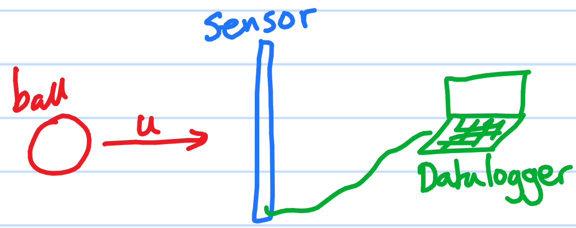
ball deforms, duration of collision



$\Delta v = v - u$

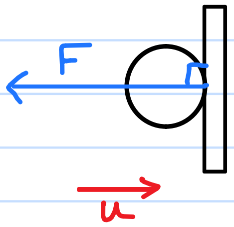


area = impulse
$= \Delta p$
$= m\Delta v$
$= m(v-u)$

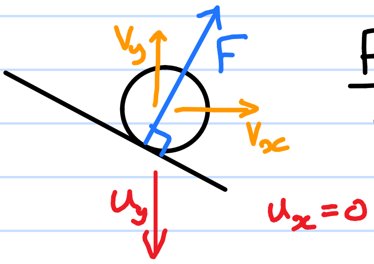


- Sensor deformed during collision
- Datalogger intepets that as force over time to produce above graph
- Impulse of ball in collision can be determined

Contact forces are always normal to tangent at point of contact between two bodies.

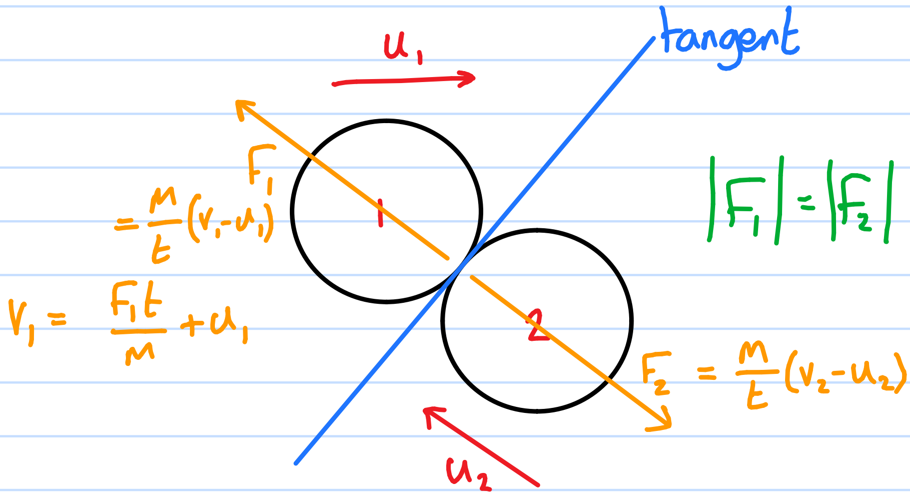


contact force on ball normal to surface



$$\frac{Ft}{m} = \underbrace{v - u}_{\text{vector difference}}$$

- Can resolve ⊥ components of force first if easier



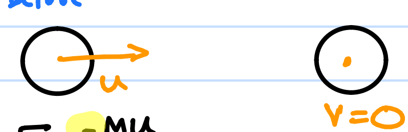
$|F_1| = |F_2|$ because of Newton's third law
" " conservation of momentum

Collide & rebound

$F = \frac{-2mu}{t}$

$\Delta p = m(v-u)$
$|u| = |v|$
$\Delta p = m(2u)$
$\Delta p = -2mu$

Collide & stop



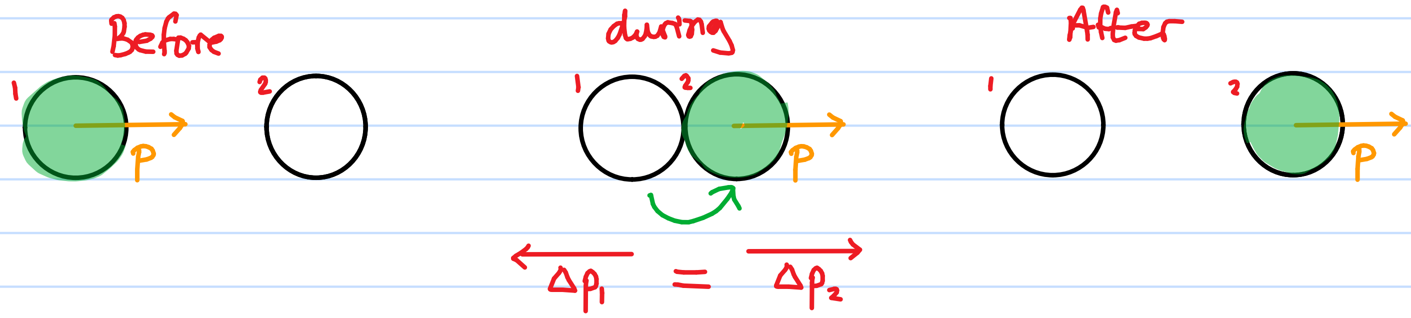
$F = \frac{-mu}{t}$

$\Delta p = m(v-u)$
$v = 0$
$\Delta p = -mu$

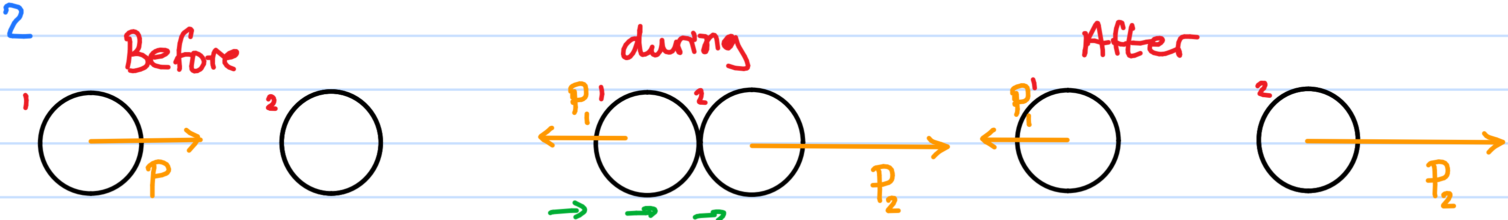
Conservation of momentum (Noether's theorem)

- Closed system = no transfer of energy or matter in or out
  Enclosed adiabatically
- Total momentum in a closed system is constant, i.e. $\frac{dp}{dt} = 0$ for closed system.
- Momentum is transfered from one body to another when the bodies interact.
- Momentum is a vector

Ex. 1



Ex. 2



$\vec{p} = \vec{p}_1 + \vec{p}_2$

($p = p_2 - p_1$ in exam)

$\overleftarrow{\Delta P_1} = \overrightarrow{\Delta P_2}$

$= p_1 - p$ $\qquad$ $= P_2 - 0$

1. Duration of collision for body 1 = for body 2
2. $|\Delta p_1| = |\Delta p_2|$
3. $F = \frac{\Delta p}{t}$
4. $\frac{\Delta p_1}{t} = \frac{-\Delta p_2}{t}$

   $F_1 = -F_2$

6 Conditions for Newton's Third Law

1. Equal magnitude ☐
2. Opposite directions ☐
3. Same line of action ☐
4. Same duration and instant ☐
5. Act between two bodies ☐
6. Same type ☐

Collisions algebraically

Collision between A & B

- $P_{A\,BEFORE} = m_A u_A$
- $P_{A\,AFTER} = m_A v_A$

$$P_{A\,BEFORE} + P_{B\,BEFORE} = P_{A\,AFTER} + P_{B\,AFTER}$$

$$\boxed{m_A \vec{u}_A + m_B \vec{u}_B = m_A \vec{V}_A + m_B \vec{V}_B}$$ is always true!

Elastic and inelastic collisions

Elastic collisions — Momentum is conserved
energy is conserved
kinetic energy is conserved (perfectly elastic)

Inelastic collisions — Momentum is conserved
energy is conserved
kinetic energy is NOT conserved
in a perfectly inelastic collision, bodies stick together, $v_A = v_B$

Perfectly Elastic

$$M_A u_A + M_B u_B = M_A V_A + M_B V_B$$ Momentum is conserved

$$\frac{1}{2}M_A u_A^2 + \frac{1}{2}M_B u_B^2 = \frac{1}{2}M_A V_A^2 + \frac{1}{2}M_B V_B^2$$ kinetic energy is conserved

Simutaneous equations

$$u_B - u_A = V_A - V_B$$

Perfectly inelastic

$$M_A u_A + M_B u_B = M_A V_A + M_B V_B$$ Momentum is conserved

$$V_A = V_B = V$$

bodies stick together, final KE as low as possible

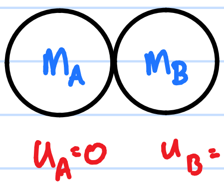
$$M_A u_A + M_B u_B = (M_A + M_B)V$$

Explosions

- Increase kinetic energy
- Energy is conserved
- Momentum is conserved

$$m_A\vec{u}_A + m_B\vec{u}_B = m_A\vec{V}_A + m_B\vec{V}_B$$

- Take the case where $u_A = u_B = 0$



$$0 = m_A\vec{V}_A + m_B\vec{V}_B$$

$$|m_A V_A| = |m_B V_B| \Rightarrow |V_B| = \frac{m_A}{m_B}|V_A|$$

$$m_A\vec{V}_A = -m_B\vec{V}_B$$